NOTE 5

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MECHANICALLY INDUCED SEA LEVEL CHANGES APPLIED TO GREENLAND'S ICE COVER DISAPPEARANCE OR DOUBLING

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INTRODUCTION

In this note I deal with a subject I looked at two decades ago. My interest had been triggered by the book over the ice times[1]. In this book it was tacitly implied that sea level variations resulting from ice shield melting or forming was uniform all over the oceans. In my mind the question arose:"Is it so, that the sea level rises more or less the same amount everywhere if important ice shields melt somewhere and does the reverse apply if big ice shields are building up somewhere on the globe?". Clearly, the Center Of Mass (COM) of the Earth is affected by this process and it was immediately obvious to me that there was more to it.

We not only have the shift of the COM, but also the orientation of the rotational equilibrium axis of the Earth (called spin axis) may undergo a slight shift if the Greenland ice melts. Even if this reorientation, known as polar wandering, is minute at global terrestrial level it may nevertheless be sufficient to influence the orientation of the *oblate* envelope of the mean ocean surfaces and cause sea level variations which are different at different latitude and longitudes and which are not everywhere negligible for us humans. These different aspects are analyzed for the specific case of Greenland by verifying what would happen if its present ice shield would either melt completely or, in contrast, double its volume. It will be shown that the mechanically induced sea level variations are not the same in different oceanic regions, including both increase and decrease. The combined consideration of these phenomena, which are only applicable to an either remote past or a far future, is likely to be novel, but the dynamical principles and the geometry behind it, are really very basic.

The subject is, of course, also interesting for people without the mathematical background implied in the quantitative derivations of the different contributions to the sea level variation. Therefore, we have inserted the next section which tries to coarsely explain "what is a center of mass" and "what is, in this context, the simple relevant behavior of the polar axis of the Earth". These properties apply to rigid (undeformable) bodies and the Earth is more or less rigid except the oceans. Exactly here is the link with the sea level which has the properties of a fluid element on one hand and continental ice shields which are rigidly bound to particular fixed geographical areas on the other hand.

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MECHANICAL REASONS FOR MEAN SEA LEVEL VARIATION

In most popular articles on sea level changes one generally assumes a simultaneous and equal rise or fall of the mean sea level all over the oceans in all circumstances. Such an assumption may be reasonable if one considers the dilatation or contraction of the seawater volume due, for instance, to a global temperature variation. Here we only consider sea level variations which have to do with the melting or formation of ice shields with a continental support, but not the global redistribution of water in the case of melting, or its global subtraction when considering ice shield formation for which we refer to the literature[2]. The case of Greenland will show that quantities of water or ice which are sufficient to change the global sea level in a measurable way are also sufficient to affect the dynamical equilibrium of the Earth and thereby require measurable and different sea surface adaptations all over the Earth.

To understand what happens we propose to perform a virtual experiment. For this purpose we consider two deep frozen water balls each with a mass of 5 kilogram whose centers are at both ends of a fully rigid but weightless rod of 10 meters length as depicted in Fig. 1.

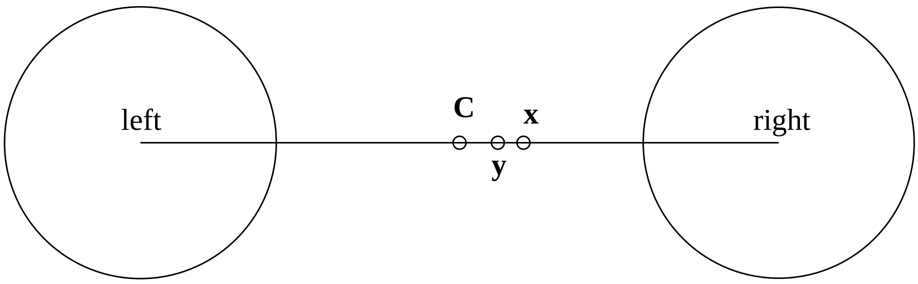

Fig. 1. Centre of mass between two ice balls of un/equal masses

The center of mass (COM) denoted by C is exactly in the middle of the rod, because measured from C the product (mass = 10 kg) times (distance to the center of the ball = 5 m) is the same on both sides. Let us now heat the left ball slightly so that exactly 1 kg of its ice melts away from all over its surface and thereafter cool the ball again. The new COM is shifted to the right of C to the point X by a distance x which is computed in the same way. The product 4 kg times (5 + x) m for the left ball must be equal to 5 kg times (5 - x) m. Basic algebra yields a shift of x = 0.555.. m. In a next step the one liter molten water is evaporated and is condensed equally on both spheres. We then end up with a left sphere of 4.5 kg and the right sphere with 5.5 kg (neglecting the difference of surface area of the left sphere which had become slightly smaller and assuming no condensation on the weightless rod). By this condensation process the COM again partly shifts to the left to the point Y by approximately 30 cm and thereby we terminate the virtual experiment. The comparison with the Greenland ice is simple. The Greenland ice melts and (forgetting a while the global redistribution of the resulting water) the COM of the Earth slightly moves in the direction of the antipode of Greenland located in the Antarctic south of Australia. The sea level will decrease by a few meters around Greenland, will almost stay constant in equatorial regions and rise around the Antarctic. But there is still the molten water which needs to be distributed over the oceans and, taken separately, this corresponds to an equal increase of the sea level all over the Earth. For the magnitude of the sea level rise/fall due to the uniform distribution of the molten Greenland ice over the oceans or alternatively in the case of sea level decrease due to the water removal, we rely on available material[2]. This is also true for the estimate of the volume of ice masses involved.

It is not really possible to describe in a few lines why the spin axis of the Earth undergoes a slight change if the Greenland ice completely melts or doubles in size. The simplest, may be, is to refer to a wooden spinning top from which a very small piece is cut off its side far from the symmetry axis. Normally, the top will still be able to spin and wobble around. The actual spin axis fixed inside the rigid wooden body will, however,have moved slightly. We mean that the top will no longer rotate exactly around the same axis we had before. The same happens with the Earth. But even if the effect is minute, the sea has to move to bring the equatorial bulge into the new position perpendicular to the new tilted spin axis. This is equivalent to slightly rotate an ellipse in a plane around its center. This ellipse is the idealized mean Earth cross section which we obtain if we cut the Earth with a plane which goes simultaneously through the previous and the new rotation axis. This effect will only be measurable at mean north or south latitudes, switching sign at the poles and the equator and hence being negligible at the poles and the equator themselves. At longitudes more or less perpendicular to the afore mentioned plane the effect vanishes. The sizes of actual polar wandering we have computed are too small to give rise to any even small climatic change.

If there is no interest in the mathematical derivations, which follow in the next three sections, one can directly go to the discussion of the predicted sea level contributions thereafter.

GREENLAND AND THE COM OF THE EARTH

We start from the assumption[2] that the volume of ice covering Greenland is approximately equal to 2.85 10^6 km^3, and 1. km^3 $=$ 10^{12} dm^3. Knowing that 1. dm^3 corresponds to the volume of a liter of normal water, the Greenland ice approximately corresponds to a mass M_g of 2.85 10^{18} kg. This figure must be compared with the approximate Earth mass for which we rely on the value[3] $M_e = 5.974\ 10^{24}$ kg.

The present COM of the Earth is put at the origin of a conventional geodetic coordinate system[4]. The x-axis of this system is in the direction of the unit vector (**i**) going through the Greenwich Meridian at latitude zero, the z-axis (**k**) points to the North Pole at a latitude of $+90^0$. The positive y-axis (**j**) crosses the equator at 90^0 East. To find an approximation of the COM of Greenland we have divided its mass into 17 big equal parts spread over the island in an approximate manner. This partitioning is described in the appendix. The geodetic latitude and longitude of the COM of the Greenland ice, obtained in this way, are 72.38^0 N and 43.38^0 W, respectively. Its distance R_g from the Earth center is 6358.71 km. The position vector $\mathbf{r_g}$ of this COM thus corresponds to

$$\mathbf{r_g} = R_g\,(\cos 43.38^0\ \cos 72.38^0\ \mathbf{i} - \sin 43.38^0\ \cos 72.38^0\ \mathbf{j} + \sin 72.38^0\ \mathbf{k}) \qquad (1)$$

We can now write down the formula for the COM of the total Earth (henceforth abbreviated by COME) consisting of two essential masses: one the Greenland ice mass M_g and the other corresponding to the rest of the Earth, namely $M_e - M_g$, whose COM position vector is represented by $\mathbf{r_e}$ with respect to the present COM, located at the origin, for the whole Earth. This very basic formula reads:

$$\mathbf{0} \,=\, M_g\,\mathbf{r_g} \,+\, (M_e \,-\, M_g)\,\mathbf{r_e} \tag{2}$$

and consequently the COM of the Earth without the Greenland ice is located at

$$\mathbf{r_e} \,=\, -\frac{M_g\,\mathbf{r_g}}{M_e \,-\, M_g} \,\approx -\frac{M_g\,\mathbf{r_g}}{M_e} \tag{3}$$

Unexpectedly, the magnitude of $\mathbf{r_e}$ is not a few mm, but 3.018 m. If we assume that the Greenland ice had suddenly disappeared (without adding the molten water to the oceans), the COME shift would occur instantaneously. In that imaginary case the sea level in the vicinity of Greenland would be 3. m above equilibrium and at the antipode (the direction of $\mathbf{r_e}$) one would be 3. m below. This situation just depicted would stay so if the Earth including the oceans was completely rigid. The new COME, however, is no longer the center of figure of the mean geoid surface, which essentially involves the oceans. There is, in fact, a layer of 3 m water at the surface of the oceans which has to move 3 m in the direction of the COME shift $\mathbf{r_e}$ in order to achieve a new equilibrium of the mean ocean surface. We know that a mass, more or less symmetrically distributed over the surface of a sphere, has its COM close to the center of that sphere, here approximately 3m from that center. For $3.0\, M_g << R_e\, M_g$, we can assume that the adaptation of the sea surface has no measurable impact onto the COME. This argument also applies in a good approximation to the water to be added or subtracted from the oceans as a consequence of the melting or forming of an ice shield somewhere on the Earth.

At an arbitrary sea level position P with the geocentric latitude λ_p and position vector $\mathbf{r_p}(\lambda_\mathbf{p})$ we may also expect a level change s, due to the COME shift. In the present case this will be limited to $-3.\,m \leq s \leq +3.\,m$. This change is a function of both the latitude λ_p and the longitude ϵ_p of the point P.

Let us briefly explain how to compute the value of s_p which corresponds to a projection of the COME translation $\mathbf{r_e}$, given in (3), onto the unit vector $\mathbf{v_p}$ representing the local vertical directed outward of the geoid. To start with, we assume that P is located onto the Greenwich meridian. The position vector there is represented by

$$\mathbf{r}(\lambda_\mathbf{0}) \,=\, a_e\,\cos\lambda_0\,\mathbf{i} \,+\, b_e\,\sin\lambda_0\,\mathbf{k} \tag{4}$$

where a_e is the equatorial and b_e the polar radius of the Earth. The subscript zero is added, to stress that we are on the Greenwich meridian where $\mathbf{r}(\lambda_\mathbf{0}) \neq \mathbf{r}(\lambda_\mathbf{p})$ but $|\mathbf{r}(\lambda_\mathbf{0})| \,=\, |\mathbf{r}(\lambda_\mathbf{p})|$. Computing the vector $\partial\mathbf{r}(\lambda)/\partial\lambda$ yields the not normalized vector along the tangent in the direction of an increasing value of λ. From this, one directly derives the value of the unit vector perpendicular to the tangent, namely $\mathbf{v_0}$, or:

$$\begin{aligned}\mathbf{v_0} \,&=\, \frac{b_e\,\cos\lambda_0\,\mathbf{i} \,+\, a_e\,\sin\lambda_0\,\mathbf{k}}{\sqrt{b_e^2\,\cos^2\lambda_0 \,+\, a_e^2\,\sin^2\lambda_0}} \\ &=\, v_{0x}\,\mathbf{i} \,+\, v_{0z}\,\mathbf{k}\end{aligned} \tag{5}$$

To bring $\mathbf{v_0}$ to the actual point P we apply a rotation around the polar axis by the angle ϵ knowing that a West longitude corresponds to a negative angle. We end up with.

$$\mathbf{v_p} \;=\; v_{0x}\,\cos\epsilon\,\mathbf{i} \,+\, v_{0x}\,\sin\epsilon\,\mathbf{j} \,+\, v_{0z}\,\mathbf{k} \tag{6}$$

The actual mean sea level change s_p at P is then simply

$$s_p \;=<\mathbf{v_p},\,\mathbf{r_e}> \tag{7}$$

where $<,>$ is employed to represent the scalar or internal product of two vectors.

Equation (3) is linear and consequently the sign of the result is simply inverted if we double the present Greenland ice shield. To be explicit, if the ice shield grows the COME moves towards Greenland and thus away from the antipode. In this case the mean sea level at equilibrium theoretically rises at least 3 m around Greenland before subtracting the sea water quantity used up by the ice shield formation.

THE MODIFICATION OF THE TERRESTRIAL INERTIA TENSOR

Based on the breakdown of the Greenland ice shield described in the appendix, we can approximate its inertia tensor N_g with respect to the geocentric co-ordinates used hitherto. On the basis of the formulae given at the end of the appendix we find:

$$N_g \;=\; 10^{32}\begin{vmatrix} 0.109184 & 0,559201\,10^{-2} & -0.237165\,10^{-1} \\ 0.559201\,10^{-2} & 0.109776 & 0.223885\,10^{-1} \\ -0.237165\,10^{-1} & 0.223885\,10^{-1} & 0.115222\,10^{-1} \end{vmatrix} \tag{8}$$

This has to be compared with the tensor applicable to the Earth[5], namely:

$$N_e \;=\; 10^{38}\begin{vmatrix} 0.8008 & 0.0000 & 0.0000 \\ 0.0000 & 0.8008 & 0.0000 \\ 0.0000 & 0.0000 & 0.8034 \end{vmatrix} \tag{9}$$

whose eigenvectors are the co-ordinate axes and the eigenvalues are here the corresponding diagonal elements. We further have to consider the inertia tensor which belongs to the water to be added to or subtracted from the oceans, which we assume to be a layer around the complete Earth considered to be sphere with radius R_m= 6371.0 km and mass M_g. The inertia of such a surface sheet around any geocentric axis is known to be equal to $w \,=\, M_g\,R_m^2$. For our problem this corresponds to an inertia matrix $N_w \,=\, w\,I_3$, where I_3 is the three dimensional unit matrix.

Let us first consider the scenario of the disappearance of the Greenland ice. The new terrestrial inertia is then given by

$$N_{no-ice} \;=\; N_e \,-\, N_g \,+\, w\,I_3 \tag{10}$$

The new spin axis inertia or largest eigenvalue of N_{no-ice} is augmented by $1.036688344\,10^{32}$ leading to a very faint despin. This means that the length of the day must have increased during ice time periods with deep glaciation, but this is out of the scope of the present analysis.

More important for us is the orientation of the new spin spin axis in original geocentric co-ordinates. The unit vector of this spin axis has the following components.

$$\begin{aligned} x_s &= 0.911849824359290\,10^{-4} \\ y_s &= -0.860792226009157\,10^{-4} \\ z_s &= 0.999999992137833 \end{aligned}$$

which corresponds to a tilt of the axis by 0.0071847^0.

This time the computations of eigenvalue and eigenvectors are, in fact, no longer linear. We thus have to verify that the difference of numbers involved are small enough to give a more or less identical but opposite result for the doubling of the Greenland ice shield. We have verified that this is the case for the spin axis resulting from the inertia tensor:

$$N_{2\times ice} = N_e + N_g - w\, I_3. \tag{10}$$

CONSEQUENCES OF A SMALL POLE WANDERING

Geographically the previous figures correspond to a pole displacement or 'wandering' of not more than 800.0 m. The tilt is a move towards Greenland on the meridian at 43.35^0 W which we call the *Greenland meridian*, now considering a full great circle which lays in the *Greenland reference plane.* At first glance the computed value may seem fully negligible, but, in fact, it is not. Let λ be an arbitrary geocentric latitude on the Greenland meridian on the Atlantic side of the Northern hemisphere before the ice had disappeared. It will then be at a latitude of $\lambda - 0.0072^0$ after the ice shield has molten. The geoid has been tilted around an axis perpendicular to the Greenland reference plane implying that the local geoid shape, which adapts to the natural oblateness of our planet, remains unchanged at the two points where this rotation axis intercepts the equator. A maximum adaptation is to be expected on the Greenland meridian itself. There an arbitrary geocentric Earth radius at $\mathbf{r_0}(x_0, y_0, z_0)$, with the original latitude λ_0, will become latitude $\lambda_0 - 0.072^0$ and consequently the geocentric Earth radius will simply vary by

$$\begin{aligned} d(\lambda_0) = {} & \sqrt{a_e^2 \cos^2(\lambda_0 - 0.0072^0) + b_e^2 \sin^2(\lambda_0 - 0.0072^0)} \\ & - \sqrt{a_e^2 \cos^2 \lambda_0 + b_e^2 \sin^2 \lambda_0} \end{aligned} \tag{11}$$

Just like in (7) for the COM adaptation, we have to project the unit vector $\mathbf{r_0}/||\mathbf{r_0}||$ onto the local vertical $\mathbf{v_0}$ multiplied by $d(\lambda_0)$ to obtain the local sea level variation, or

$$s_0 = d(\lambda_0) \frac{< \mathbf{r_0},\, \mathbf{v_0} >}{||\mathbf{r_0}||} \tag{12}$$

Let us now turn to an arbitrary point P, not located on the Greenland meridian with the geocentric position vector $\mathbf{r_p}(x_p, y_p, z_p)$, the latitude λ_p and the longitude ϵ_p. In that case the geometry gets slightly more involved. This is due to the fact that the virtual wandering of the point P is not a rotation but a redefinition of a fixed geocentric position in a new slightly tilted ellipse with semi-major and minor axis a_p and b_p contained in a plane parallel to the Greenland reference plane. This is partially depicted in Fig. 2, which represents the terrestrial equator with the projection of the Greenland reference plane and the corresponding parallel plane containing P. We also introduce an alternative geocentric co-ordinate system, shown in the figure, in which the new x'-axis is no longer linked to the Greenwich meridian but now to the perpendicular to Greenland reference plane on the Atlantic side or at $90^0 + 43.38^0$ W. This corresponds to a rotation with an angle $\beta = -133.38^0$ around the z-axis, which itself remains unchanged. It will thus be sufficient to replace ϵ_p by $\epsilon'_p = \epsilon_p - \beta$ to obtain x'_p, y'_p, z'_p in the new coordinate system. Next we have to characterize the projected ellipse contained in between the points C and D on the equator, noting that a_p is equal to half the length of the line $\overline{\mathrm{CD}}$. The length Nb is equal to x'_p and consequently $a_p = \sqrt{a_e^2 - x'^2_p}$.

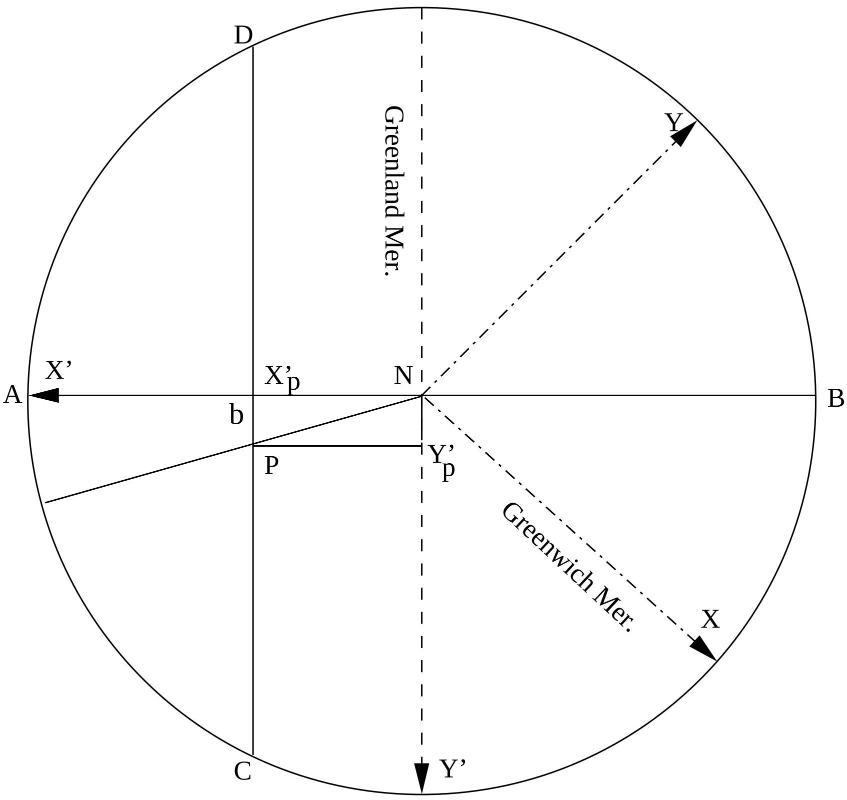

Fig. 2 Equatorial projection of the point P moved on an ellipse in the plane with projection CD.

On the other hand, the value of the semi-minor axis is the length of the perpendicular on the equator at the point b up to the circumference of the ellipse going through the spin axis at N, thus corresponding to (4) where the (so far unknown) angle α has to replace λ_p. Thereby, we observe that $x'_p = a_e \cos\alpha$, providing the value of α. Hence, $b_p = b_e \sin\alpha$ is defined. Finally, we have to determine the angle γ in the smaller ellipse, with the yet known major axes a_p and b_p. The angle γ is enclosed by the vector perpendicular on $\overline{\mathrm{AB}}$ to the point P inside the plane of the smaller ellipse on the one hand and the semi-major axis a_p of this ellipse on the other hand. From Fig. 2 we see immediately that $y'_p = a_p \cos\gamma$. We further know that $z'_p = b_p \sin\gamma$ defining γ uniquely by $\gamma = \arg(z'_p/b_p, y'_p/a_p)$. The change of the distance to the rotation axis AB is then

$$d'(\lambda_p, \epsilon_p) = \sqrt{a_p^2 \cos^2(\gamma_p - 0.0072^0) + b_p^2 \sin^2(\gamma_p - 0.0072^0)} - \sqrt{a_p^2 \cos^2\gamma_p + b_p^2 \sin^2\gamma_p} \tag{13}$$

This yields a level change whose direction is perpendicular to the rotation axis AB and

Table 1.Equilibrium induced sea level changes in the Northern Hemisphere due to the melting of Greenland's present ice shield.

City	Country	latit.^{0}	longit.^{0}	Δ_{COM}	$\Delta_{p.w.}$	Total
Nordvik	Russia	73.53	110.28	-2.75	0.13(1.46)	-2.62
Barrow	US	71.28	-156.78	-2.74	-0.65(1.64)	-3.39
Hammerfest	Norway	70.65	23.82	-2.73	0.65(1.68)	-2.08
Iqaluit	Canada	63.72	-68.50	-2.55	1.93(2.14)	-0.62
Bergen	Norway	60.38	05.32	-2.51	1.53(2.31)	-0.98
Amsterdam	Netherlands	52.35	04.92	-2.29	1.73(2.60)	-0.56
Vancouver	Canada	49.25	-123.12	-2.26	0.48(2.66)	-1.78
Boston	US	42.35	-71.05	-1.87	2.23(2.68)	0.36
Kushiro	Japan	42.97	144.45	-1.86	-2.66(2.68)	-4.52
Lisbon	Portugal	38.72	-09.12	-1.76	2.17(2.62)	0.41
Shanghai	China	31.00	121.40	-1.34	-2.29(2.37)	-3.63
New Orleans	US	29.95	-90.07	-1.44	1.60(2.32)	0.16
Taipeh	Taiwan	25.03	121.52	-1.07	-1.99(2.05)	-3.06
Karachi	Pakistan	24.87	67.03	-1.33	-0.71(2.04)	-2.04
Havana	Cuba	23.12	-82.35	-1.09	1.51(1.94)	0.42
Honolulu	US	21.30	-157.85	-1.17	-0.75(1.82)	-1.92
Acapulco	Mexico	18.87	-98.42	-0.99	0.94(1.64)	-0.05
Madras	India	13.07	80.27	-0.72	-0.66(1.18)	-1.38
Paramaribo	Suriname	05.82	-55.17	-0.01	0.53(0.54)	0.52
Singapore	Singapore	01.28	103.85	-0.03	-0.10(0.12)	-0.13

consequently we have to use the normalized direction $\mathbf{r_p} = a_p \cos\gamma\ \mathbf{j'} + b_p \sin\gamma\ \mathbf{k'}$. The local vertical $\mathbf{v_p}$ is to be computed with respect to a meridian going through P and the North pole. The actual size of corresponding component of the sea level change is

$$s_p = d'(\lambda_p, \epsilon_p)\, \frac{< \mathbf{r_p}, \mathbf{v_p} >}{||\mathbf{r_p}||} \tag{14}$$

If λ_0 in (11) and γ_p in (13) are positive latitudes, we have $0 < d(\lambda_0)$ and $0 < d'(\lambda_p, \epsilon_p)$, provided the longitude referred to is within 90^0 from the Greenland meridian. This follows from the following first order development in function of a small positive ϵ:

$$\begin{aligned} d^2(\epsilon) &= a^2 \cos^2(\lambda - \epsilon) + b^2 \sin^2(\lambda - \epsilon) \\ &\approx d^2(0) + \epsilon\,(a^2 - b^2) \sin 2\lambda \end{aligned}$$

provided $b < a$, what is the case in our application. It is thereby conspicuous that the values of d have an extremum at latitudes of $\pm 45^0$ and are close to zero at the poles and the equator.

NUMERICAL RESULTS

Summarizing, we have first obtained a COM shift of approximately 3m in the direction of the Antarctic South of Australia if the present Greenland ice melts. Second, we have found out that in these circumstances the North pole moves 0.0072^0 towards Greenland. If, in contrast, the ice cover would double its size the previous values apply in a very good approximation but with inverted directions. The sea level change, due to the COM shift, is computed with respect to the Earth center and has to be projected onto the local vertical of a specific geographic place. This is explained in the third section and the actual projections are represented by Δ_{COM} in the tables 1 above and 2 hereafter.

The effect of the pole wandering is geometrically more involved and the way to compute the projections on the local vertical, represented by $\Delta_{p.w.}$ in table 1 and 2, is presented in the previous section. The projection as such always leads to smaller figures than those predicted theoretically and not taking the local vertical into account.

Table 2. Equilibrium induced sea level changes in the Southern Hemisphere due to the melting of Greenland's present ice shield.

City	Country	latit.^{0}	longit.^{0}	Δ_{COM}	$\Delta_{p.w.}$	Total
Ushuaia	Argentina	-54.78	-68.28	2.40	-2.30(-2.53)	+0.10
Stanley	Falklands	-51.70	-57.85	2.33	-2.53(-2.62)	-0.20
Dunedin	New Zealand	-45.70	170.50	2.10	2.23(-2.69)	+4.33
Punta Delgada	Argentina	-42.77	-63.62	2.05	-2.51(-2.68)	-0.46
Hobart	Australia	-42.92	147.32	2.06	2.63(-2.68)	+4.69
Wellington	New zealand	-41.28	174.77	1.92	2.10(-2.66)	+4.02
Buenos Aires	Argentina	-34.58	-58.67	1.76	-2.42(-2.51)	-0.66
Cape Town	South Africa	-33.92	18.42	1.52	-1.18(-2.49)	-0.34
Santiago	Chile	-33.45	-70.67	1.68	-2.20(-2.47)	-0.52
Brisbane	Australia	-27.50	153.02	1.46	2.11(-2.20)	+3.57
Curitiba	Brazil	-25.42	-49.25	1.41	-2.07(-2.08)	-0.66
Port Louis	Mauritius	-20.15	57.48	0.83	0.33(-1.74)	+1.26
Papeete	Polynesia	-17.52	149.57	1.03	1.50(-1.54)	+2.53
Lima	Peru	-12.05	-77.03	0.69	-0.91(-1.10)	-0.22
Luanda	Angola	-08.88	13.23	0.35	-0.45(-0.81)	-0.10
Jakarta	Indonesia	-06.17	106.82	0.42	0.50(-0.57)	+0.92
Belém	Brazil	-03.92	-69.62	0.33	-0.33(-0.37)	+0.00

All towns selected in tables 1 & 2 are located on - or are very near to a coast line. The break down in North and South hemispheres shows the systematic effect of latitude on the value of Δ_{COM}. The variation $\Delta_{p.w.}$ is more versatile. To make the effect of longitude visible, we have added the value of $\Delta_{p.w.}$ in brackets, representing the case where the town of a given row would be located on the Greenland meridian (at Western longitude). The latter numbers clearly show the extrema at + and -45^0 latitude and the vanishing contribution on the equator.

The columns with the totals do not consider the 7.3 m general sea level increase[2] due to the distribution of the water resulting from the molten Greenland ice shield. Although the figures of table 1 & 2 are not negligible, they are not able to offset these $7.3m$. On the contrary, in the case of sea level rising, they may aggravate the situation especially for New Zealand and around the South-East of Australia and Tasmania. In the opposite case (the more probable for the next 10,000 years), with formation of new and the increase of existing ice shields in the high North, the sea level will thus decrease all over the world, but not everywhere by the same amount. It looks as if we presently are in a period of high sea levels. The 'quick' melting of the Greenland ice, should it further dramatically accelerate, will at any rate require at least many centuries as Earth observation satellites of ESA and NASA seem to confirm. Thus more than time enough for mankind to adapt, as it did ever since historical times when living conditions changed in densely populated areas in coastal regions for whatever unescapable reasons.

More interesting than unreasonable panicking is to look into the paleolontological past, for which polar wanderings of many degrees have been reported to have occurred in very remote periods of Earth's history. The analytical formula for the sea level change experienced at 45^0 colatitude (the extremum) from the original North pole – on the meridian containing the old and the new pole – for hypothetical pole wanderings denoted by τ can be derived from (11). We get

$$\begin{aligned} d_\tau(45^0) \;=\; & \sqrt{a_e^2\cos^2(45^0-\tau)\,+\,b_e^2\sin^2(45^0-\tau)} \\ & -\;\sqrt{a_e^2\cos^2 45^0\,+\,b_e^2\sin^2 45^0} \end{aligned}$$

To compute the angle between the local vertical and geodetic position vector, namely $\mathbf{v_0}$ and $\mathbf{r_0}$ we first introduce the shorthand $a_n = \sqrt{a_e^2 + b_e^2}$. Let us write c for either cos or $\sin 45^0$ and consider the x, z-plane to construct the necessary two dimensional vectors where the geodetic direction is given by $\mathbf{r_0}(ca_e, cb_e)$, the not normalized tangent at geodetic latitude 45^0 has the components $(-ca_e, cb_e)$ and finally the outward directed local vertical has the components $(b_e/a_n, +a_e/a_n)$. Performing the normalization of $\mathbf{r_0}$ yields in this particular case a value of s_0 in (12) now equal to

$$s_\tau(45^0) \;=\; d_\tau(45^0)\frac{2\,a_e\,b_e}{a_e^2\,+\,b_e^2} \tag{16}$$

The angle θ between the local vertical and the geodetic position vector is, in the specific case of a latitude of 45^0, independent of τ and can be found from $\cos\theta = 2a_eb_e/(a_e^2+b_e^2)$. Its value is equal to 0.99999436, corresponding to an angle $\theta = 0.19^0$, thus altogether of no influence upon the result of (16). The value of $d_\tau(45^0)$ is a fairly linear function of τ for $|\tau| < 5^0$ and at 5^0 we would have the cataclysmic value of a sea level change of 1866.0 m or almost 2. km. For a pole wandering of one degree we still find 373.24 m and if the meridian of wandering would move towards Italy, for instance, the pole shift could dry out most of the Mediterranean. Thus spin axis wandering may be at the origin of large sea level changes at mid latitudes. Nevertheless tilt angles larger than a few tenths of a degree are improbable if we see that the Greenland ice is causing a shift of not more than 0.0072^0. The large pole wanderings reported in the literature can only be understood physically if they were the result of a multitude of small steps.

APPENDIX

To model the basic dynamical parameters of Greenland we have subdivided the island into 17 points, each with the same mass. This selection is largely arbitrary. Each point is defined by its North Latitude and West longitude as tabulated hereafter.

Table 3. Position definition of the breakdown of the Greenland ice mass.

	lat. N	long. W		lat. N	long. W		lat. N	long. W
1	62.0	47.0	2	64.0	48.0	3	66.0	48.0
4	68.0	45.0	5	68.0	35.0	6	70.0	46.0
7	70.0	33.0	8	72.0	45.0	9	72.0	35.0
10	74.0	48.0	11	74.0	35.0	12	76.0	52.0
13	76.0	35.0	14	78.0	56.0	15	78.0	35.0
16	80.0	62.0	17	80.0	35.0			

On the the basis of the latitude λ_j and longitude ϵ_j for a point j, with $1 \leq j \leq 17$, we compute the unit vector $\mathbf{u_j}\,(x_j, y_j, z_j)$. The distance from the Earth center to th point j is set equal to

$$R_j \;=\; \sqrt{a_e^2 \cos^2 \lambda_j \;+\; b_e^2 \sin^2 \lambda_j} \;+\; 1.25\,\text{km}$$

where a_e is the mean equatorial Earth radius which we set equal[4] to 6378.14 km and b_e is the polar Earth radius equal to 6356.75 km. For the masses of the 17 points are assumed equal, the corresponding position vector $\mathbf{r_g}$ of the point mass representing Greenland as a whole is given by

$$\mathbf{r_g} \;=\; \frac{1}{17} \sum_{j=1}^{17} R_j\, \mathbf{u_j}$$

The corresponding geodetic polar co-ordinates of $\mathbf{r_g}$ are

$$R_g \;=\; 6358.71\,\text{km}, \quad \lambda_g \;=\; 72.3798^0, \quad \epsilon \;=\; -43.3770^0.$$

The elements of the Inertia tensor or equivalently the 3×3 matrix N_g, representing the approximate contribution of the present Greenland ice to the rotational dynamics of the Earth, is obtained by computing

$$n_{xx} = \frac{M_g}{17} \sum_{i=1}^{17} (y_i^2 + z_i^2), \qquad n_{yy} = \frac{M_g}{17} \sum_{i=1}^{17} (x_i^2 + z_i^2), \qquad n_{zz} = \frac{M_g}{17} \sum_{i=1}^{17} (x_i^2 + y_i^2),$$

$$n_{xy} = -\frac{M_g}{17} \sum_{i=1}^{17} x_i\, y_i = n_{yx}, \quad n_{xz} = -\frac{M_g}{17} \sum_{i=1}^{17} x_i\, z_i = n_{zx}, \quad n_{yz} = -\frac{M_g}{17} \sum_{i=1}^{17} y_i\, z_i = n_{zy}.$$

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